An Ultra-High Quality Factor Microwave Sapphire Loaded Superconducting Cavity Transducer

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Abstract

The performance of a prototype ultra-high quality factor ($Q > 10^8$) Sapphire Loaded Superconducting Cavity Transducer (SLSCT) operating at X-band is described. The measured displacement noise floor of $3.0\pm0.6\times10^{-16}$ m/√Hz (~2×10^{-11} g/√Hz) at 1 kHz was an order of magnitude better than a similar room temperature transducer, and several orders of magnitude better than current commercially available accelerometers. Improvements in the transducer's microwave system should enable the noise floor to be lowered by several orders of magnitude. An electromagnetic model of the transducer has also been developed which permits the tuning coefficient, and hence the displacement sensitivity to be accurately predicted.

Introduction

The quest to sense the minute motions necessary to detect gravitational radiation has resulted in the development of several different ultra-sensitive motion transducers [1]. At the University of Western Australia (UWA) the quest has led to the development of the SLSCT. The low loss transducer has the highest electrical quality factor ($Q > 10^8$) of any previously constructed transducer. Even at room temperature the losses are low enough that a sapphire transducer out performs the currently commercially available accelerometers.

Table 1 compares the performance of the SLSCT when operating cryogenically, at room temperature and with typical commercially available devices. In this paper we report on the expected improvements to the current sensitivity of the SLSCT, showing that a displacement sensitivity of ~10^{-19} m/√Hz could be achieved.

Transducer	Displacement Sensitivity [m/√Hz]	Acceleration Sensitivity [g/√Hz]
Sapphire Transducer @ T = 300 K	3×10^{-15}	2×10^{-10}
SLSCT @ T = 4.2 K	3×10^{-16}	2×10^{-11}
Commercial @ T = 300 K	10^{-12}	6×10^{-9}
Projected SLSCT @ T = 4.2 K	10^{-19}	6×10^{-15}

Table 1. *Comparing the sapphire transducer in several different forms with a typical commercial transducer. The projected sensitivity of the SLSCT is also shown and will be discussed in the paper.*

Transducer Construction

The dielectric part of the transducer consisted of two short cylindrical sapphire rods or "mushrooms". One mushroom was attached to a niobium membrane as shown in *figure 1* which formed the top of a superconducting niobium cavity. The other mushroom was rigidly attached to the base of the cavity. The superconducting niobium cavity acts as a shield to eliminate radiation loss and maximise the electrical Q in the sapphire dielectric system.

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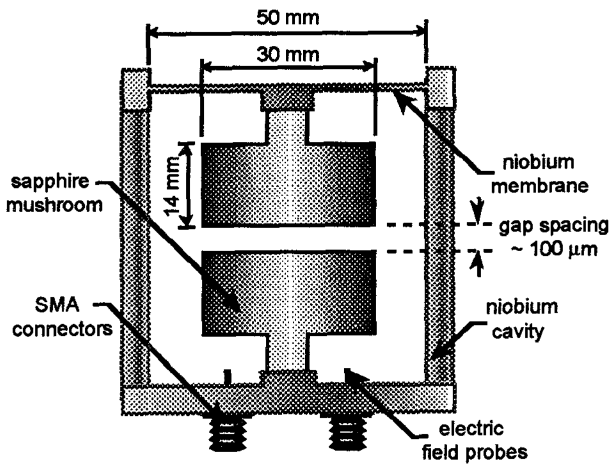

Figure 1. The two sapphire mushrooms enclosed within the superconducting niobium cavity. Two microwave probes allow the transducer to be operated in both transmission and reflection.

The motion of the niobium membrane alters the distance between the two mushrooms. This modulates the resonant frequency of the sapphire resonator. The modulated microwave signal is then demodulated using the microwave readout circuit shown in *figure 2.* We used an X-band microwave pump signal to excite the high azimuthal order *whispering gallery* (WG) modes ($TM_{6,1,1}$, $TM_{7,1,1}$, $TM_{8,1,1}$), as they possessed the highest tuning coefficients and electrical quality factors.

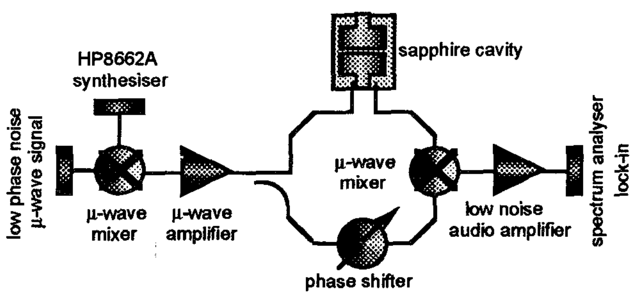

Figure 2. The microwave readout showing the sapphire dielectric resonator and demodulation circuit. A low phase noise pump signal is provided by a Sapphire LOaded Superconducting Cavity (SLOSC) oscillator [3]. The signal is mixed with a low phase noise HP8662A frequency synthesiser to provide a tunable pump signal. The signal is amplified, and injected into the dielectric cavity. The modulated output signal is then demodulated using a microwave phase bridge, which combined with the dielectric cavity form a frequency discriminator characterised by a sensitivity dV/df [Volts/Hz].

The sensitivity of the transducer can be calculated from the known parameters of both the dielectric resonator system and the microwave readout. The parameters of importance are the frequency discriminator conversion ratio *dV/df,* which is a function of the offset frequency, and the tuning coefficient *df/dx.* These can be combined to give an expression for the transducer's sensitivity,

$$\sqrt{S_x} = \sqrt{S_v}\,\frac{1}{dV/df}\,\frac{1}{df/dx} \quad [\mathrm{m}/\sqrt{\mathrm{Hz}}] \tag{1}$$

where the S_v is the spectral density of the voltage measured at the output of the readout circuit.

The dielectric section of the transducer was cooled to a temperature of 4.2 K, using a cryogenic dewar as shown in *figure 3.* It was necessary to attach the dielectric section to the base of a four-stage passive vibration isolation system to allow the transducer to be operated within its position bandwidth (~10^{-11} m).

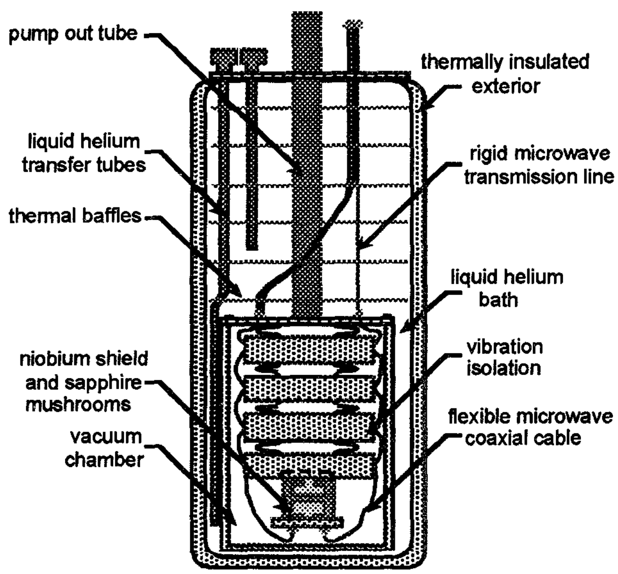

Figure 3. The cryogenic dewar showing the vacuum chamber, and surrounding liquid helium bath. A four-stage passive vibration isolation system isolates the transducer from external vibration. Thin, flexible coaxial cables allow the microwave signal to be delivered to the Sapphire Loaded Cavity, without degrading the vibration isolation.

Phase Noise

Theoretical modelling of the noise characteristics of the transducer have shown that the sensitivity of the transducer is limited by the pump signal's phase noise. To generate a frequency tunable signal we were constrained to use a HP8662A frequency synthesiser mixed with the output of a low phase noise SLOSC oscillator. In this system the HP8662A's phase noise dominates, limiting the phase noise to -125 dBc/Hz at 1 kHz. This phase noise translates into an equivalent displacement noise through the expression,

$$\delta x(\omega_m) = \frac{\omega_m}{2\pi} \frac{1}{df/dx} \sqrt{S_\phi(\omega_m)} \qquad (2)$$

where ω_m is the angular frequency of the mechanical oscillator (1 kHz), df/dx is the tuning coefficient (~2 MHz/μm) and $S_\phi(\omega_m)$ is the spectral density of the pump signal's phase noise. With this level of phase noise the displacement sensitivity is limited to $\sim 3\times10^{-16}$ m/√Hz at 1 kHz. This is equivalent to the measured displacement sensitivity at 1 kHz.

The application of recently developed ultra-low phase noise microwave oscillators at UWA [4], shall enable the displacement sensitivity to be increased by an order of magnitude as shown in *figure 4*. The increase in sensitivity comes with a degradation of the frequency tunability, the pump oscillator must be closely tuned to the operational frequency of the transducer. It is possible that this performance can be achieved given the constraints of the tuning. In this case, the approach will be to design the pump oscillator to match that of the transducer [7].

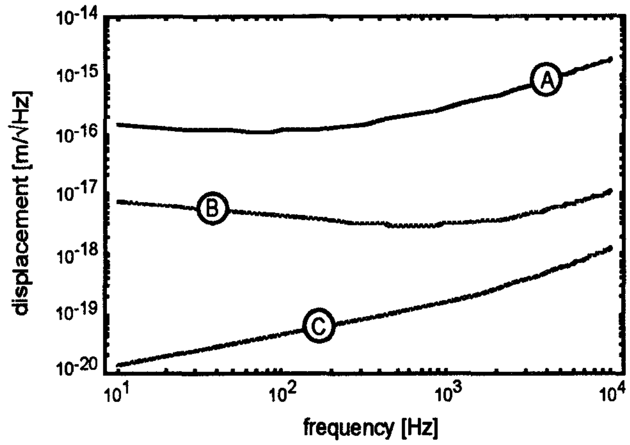

Figure 4. *The displacement sensitivity as limited by the pump signal's phase noise. Curve A shows the achievable displacement sensitivity using the current frequency tunable microwave system. Curve B shows the expected sensitivity using a liquid nitrogen cooled sapphire oscillator with phase noise of -165 dBc/Hz at 1 kHz [4].Curve C shows the displacement sensitivity if the proposed 20dBc/Hz improvement can be made to the liquid nitrogen cooled sapphire oscillator [6].*

Electromagnetic Modelling

To improve our understanding of the tuning coefficient of the transducer and provide a direction for future improvement, we applied the mode matching method to solve for the resonant frequencies as a function of the gap spacing between the sapphire mushrooms. In particular we were interested in modelling the $TM_{m,1,1}$ (where $m > 6$) whispering gallery modes, as these modes were found to possess high tuning coefficients and electrical Q's. *Figures 5* and *6* show the theoretical and experimental results for the $TM_{7,1,1}$ and $TM_{8,1,1}$ modes. Very good agreement is found between the experimentally measured tuning coefficients and the theory. For some modes, small discrepancies existed between the theoretical and experimental resonant frequencies at zero gap spacing, these were attributed to the uncertainties in dielectric permittivity and physical dimensions of the sapphire mushrooms.

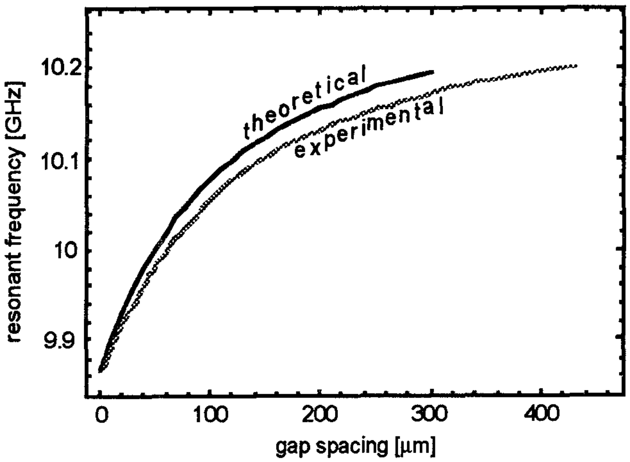

Figure 5. The resonant frequency for the $TM_{8,1,1}$ mode as a function of gap spacing between the sapphire mushrooms. As the gap spacing increases the discrepancy between theory and experiment becomes significant.

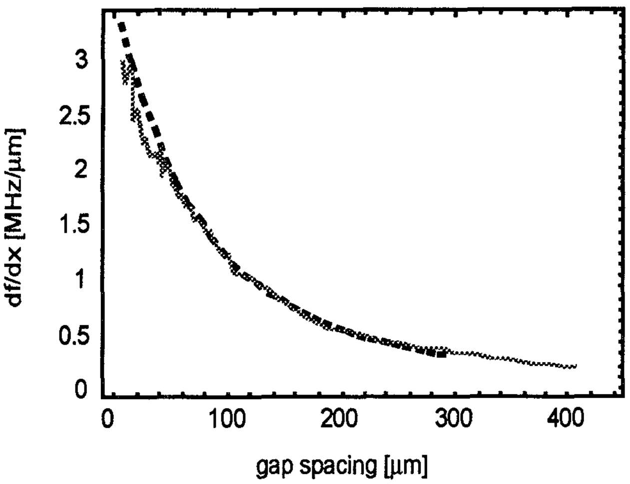

Figure 6. The tuning coefficient df/dx for the $TM_{8,1,1}$ mode as a function of gap spacing between the sapphire mushrooms. Comparison with the experimental data and theory shows that the model can accurately predict the tuning coefficient even at large gap spacings.

Conclusion

The displacement sensitivity of the SLSCT ($3.0\pm0.6\times10^{-16}$ m/√Hz) has been shown to be limited by the phase noise from the microwave pump. With the recent advent of ultra-low phase noise microwave oscillators at UWA [4,5,6], it is possible to improve the sensitivity of the transducer by at least a further order of magnitude [3,4,5].

The high accuracy of the electromagnetic model also allows the tuning coefficient to be predicted with confidence, and this will aid in accurately calibrating the transducer in the future.

References

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